

## Problem G Goal-line Technology

The 2022 FIFA World Cup is ongoing in Qatar. This year, a lot of new technologies are used to assist referees in eliminating most controversial decisions.



Figure G.1: World Cup 2010 incident

English fans will never forget the incident in the match against Germany in 2010. The shot of Frank Lampard made the ball cross the goal-line but the referees did not realize it and did not give a goal to England. This incident also led to the development of the goal-line technology. This technology determines whether the whole of the ball has crossed the goal-line.

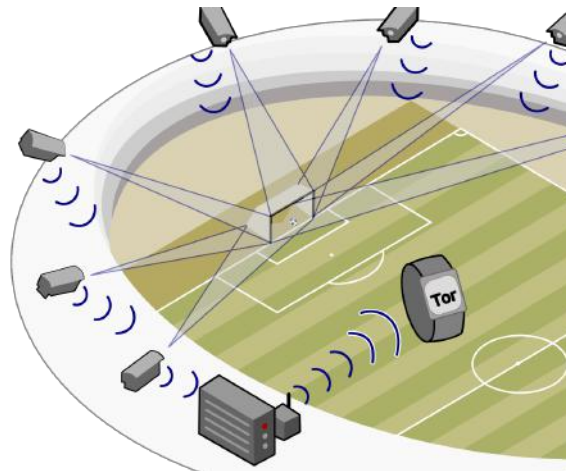


Figure G.2: Goal-line Technology Settings

The technology consists of 14 high-speed cameras mounted around the stadium. The high-speed cameras track the ball with high accuracy and use *triangulation* to calculate its precise position relative to the goal-line. Triangulation is a geometric technique of calculating the distance and position to and of, respectively, an unknown point with the help of two known points. As the name suggests, the system forms a triangle between these three points and uses the angles between them to determine the whereabouts of the third unknown. The system software then creates a 3D image of the ball relative to the line.

In the football law, it is stated that:

A goal is scored when the whole of the ball passes over the goal-line, between the goalposts and under the crossbar, provided that no offence has been committed by the team scoring the goal.

Mathematically speaking, consider the top-down projection of the ball and the goal. The ball should be a circle  $B$ . The goal-line is the area  $S$  bounded by 2 parallel lines  $x = x_1$  and  $x = x_2$  ( $x_1 \neq x_2$ ). These 2 lines split the whole plane into 3 parts:

- The *goal-line* is the area bounded by these 2 lines;
- The *in-goal side* is the area adjacent to line  $x = x_1$ ;
- The remaining one is the *in-play side*.

It is a goal if there is a moment where the ball is completely inside the *in-goal side*. In other words, these conditions hold:

- The common area of  $B$  and the *goal-line* is 0;
- The common area of  $B$  and the *in-play side* is 0.

In this problem, we only care about the relative position of the ball to the goal line, thus we only consider the  $x$  coordinate and ignore the  $y$  and  $z$  coordinates. The movement of the ball was tracked by the 14 cameras during some time-frames, resulting in a list of  $n$  coordinates  $p_1, p_2, \dots, p_n$  where  $p_i$  is the  $x$  coordinate of the center of the ball captured at the  $i$ -th frame.

You are given the list  $p$ , the radius  $r$  of the ball, the position of the goal-line. Your task is to determine if it is a goal.

## Input

- The first line consists of 4 integers  $n, r, x_1, x_2$  ( $1 \leq n \leq 10^4, 1 \leq r \leq 111, |x_1| \leq 10^6, |x_2| \leq 10^6, x_1 \neq x_2$ ).
- The second line consists of  $n$  integers  $p_1, p_2, \dots, p_n$  ( $|p_i| \leq 10^6$ ).

## Output

You should print GOAL if it is a goal, and print NO GOAL otherwise.

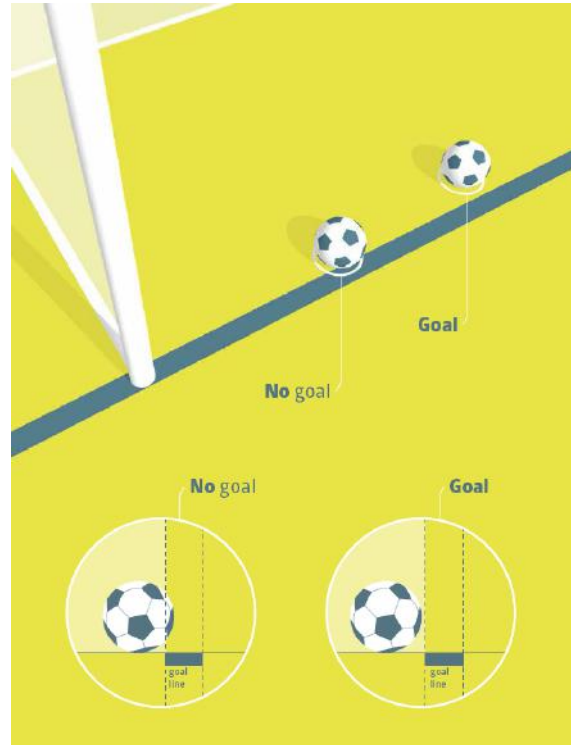
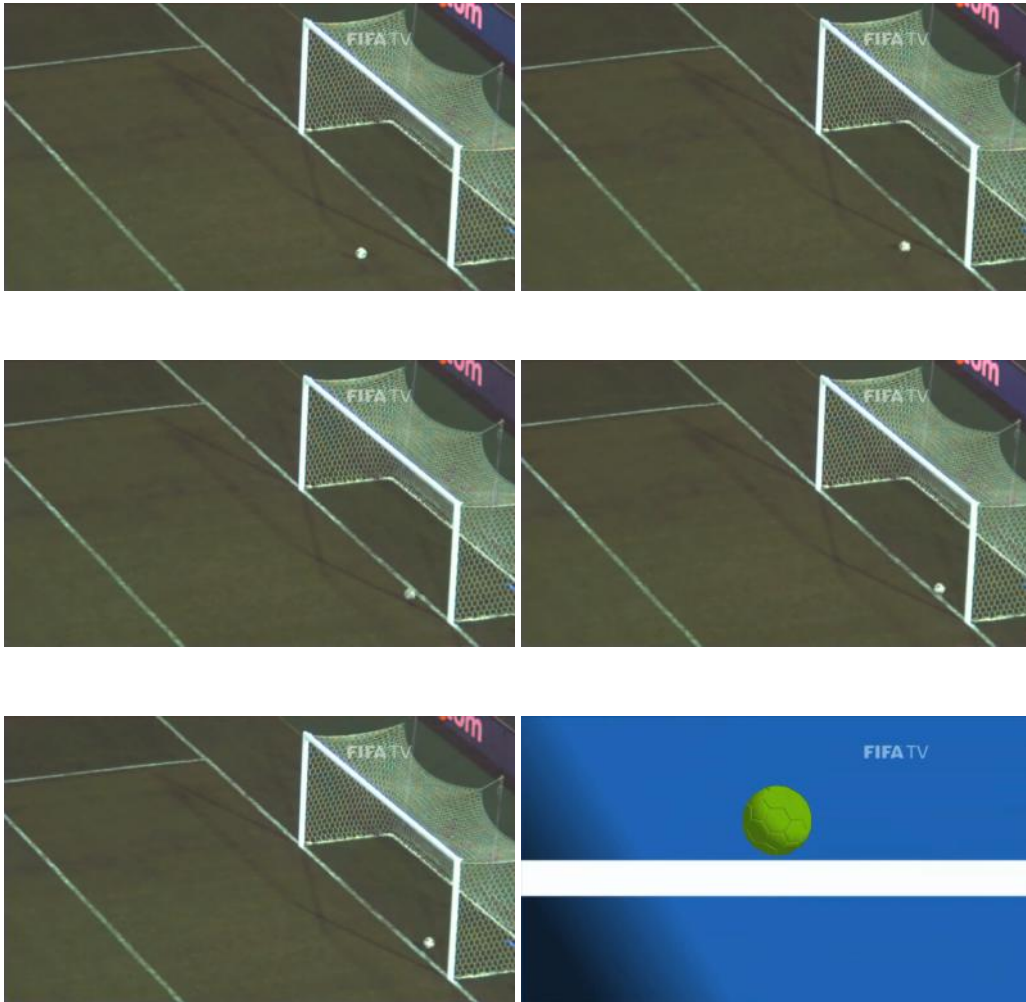


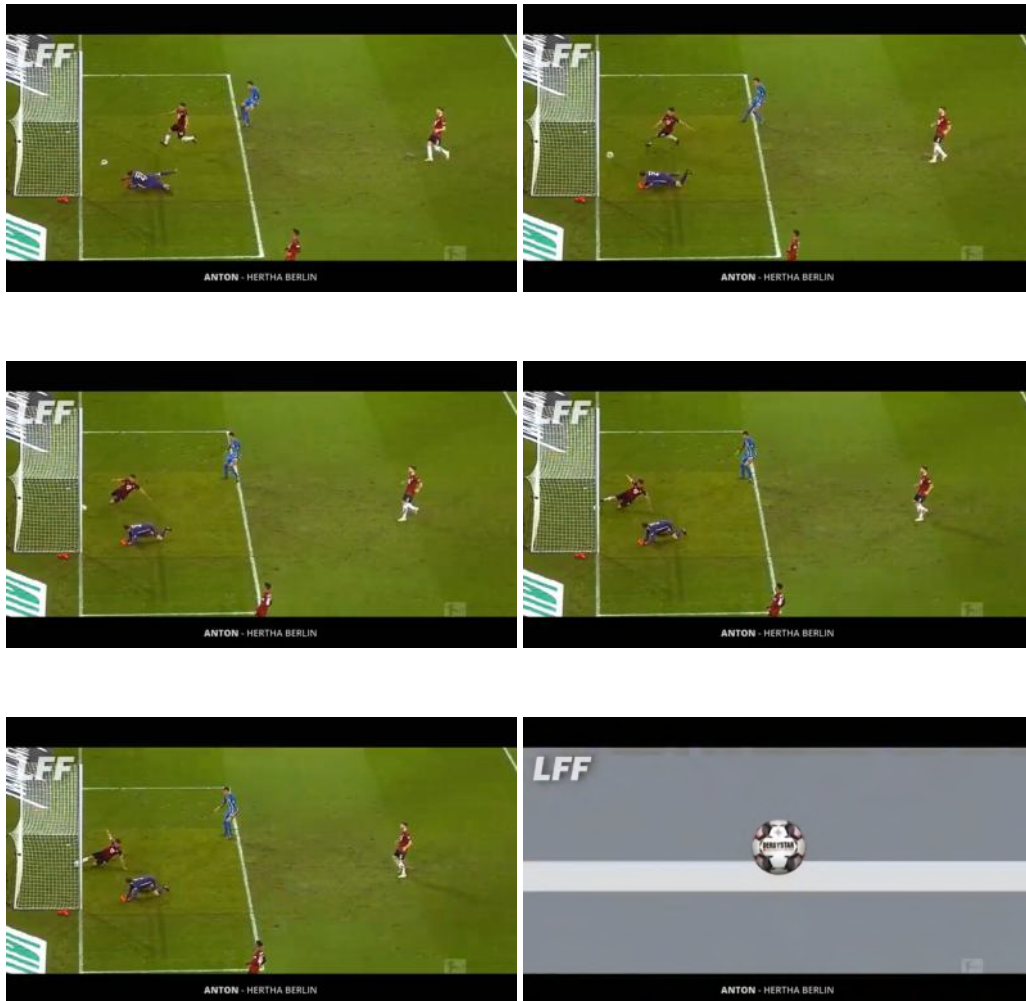
Figure G.3: No Goal vs Goal

## Explanation of the samples

In the first sample, the ball slowly rolls into the goal.



In the second one, the ball rolls into the empty goal but a defender has an excellent clearance.



In the last one, the ball does not move but it is inside the goal.

### Sample Input 1

```
6 10 120 110
90 100 110 120 130 140
```

### Sample Output 1

GOAL

### Sample Input 2

```
8 10 0 10
10 4 -2 -8 -1 5 10 15
```

### Sample Output 2

NO GOAL

### Sample Input 3

```
1 1 2 3
1
```

### Sample Output 3

GOAL