

The 2022 ICPC Asia Ho Chi Minh Regional Contest

HCMUTE – 9 December 2022

Problem E Emotional Damage

Steven just got a dice as his birthday gift! He likes it a lot, and often uses it to play games with his friends. But one day, he could not find his dice anywhere! He asked his dad for help:

- Dad, do you see my dice anywhere?

- No, I don't. But you can use mine. Here!

Steven was very happy to have his dad's dice. But he quickly became very confused when he found out that his dad's dice was not normal. The dice can be seen as a 2D convex polygon of n vertices. The polygon is not even regular, making Steven more confused.

- Dad, why are the dice sides not the same? Are you sure we can play with this dice?

- Looks son. Back in my day, dice was not invented yet. When I went to school, I had to walk 20 miles. Uphill. Both ways. 26 hours a day. On one foot. My other foot was doing all the probability calculation! You'll have to figure it out yourself.

So Steven needs to know how *fair* is the dice. To do that, he wants to know, for each side of the dice, what is the probability of that side being the landing side when the dice is rolled.

Formally, when rolling a dice, the dice will be lifted high enough above the ground. Then, we choose an angle α uniformly and rotate the dice by the α about its *center of gravity*. Finally, we let the dice fall.

When the dice hits the ground, the dice could be in a *non-stable* state. The dice will continue rolling until it becomes stable. Assuming:

- The dice has uniform mass density, its *center of gravity* will be the *arithmetic mean* position of all the points inside it,
- The mass of the dice is big enough, such that there is no bouncing on contact, and the dice will not roll any further when it is already stable.
- The ground is flat.

Consider the project H of the *center of gravity* of the dice to the ground:

- If *H* lies on one of the dice sides, the dice is *stable*.
- If *H* lies to the left of any contact point of the dice with the ground, the dice will roll to the left.
- If *H* lies to the right of any contact point of the dice with the ground, the dice will roll to the right.

Given the polygon of n vertices, representing the dice. Please help Steven find out how *fair* the dice is, by finding out the probability of each side of the dice being the landing side.



icpc.foundation



The 2022 ICPC Asia Ho Chi Minh Regional Contest

HCMUTE - 9 December 2022



Input

The first line contains a single integer $n~(3 \le n \le 10^5)~$ – the number of vertices of the dice.

The *i*-th line of the next *n* lines contains two integer x_i and y_i $(-10^9 \le x_i, y_i \le 10^9)$ – the coordinates of the *i*-th vertex of the dice.

It is guaranteed that:

- the points are listed in counter-clockwise order,
- the given polygon is convex,
- no three consecutive vertices of the polygon lie on the same line

Output

Output n lines. On the *i*-th line output the probability of the *i*-th side being the landing side. The *i*-th side is formed by the *i*-th and the $((i \mod n) + 1)$ -th vertices in the order given by the input.

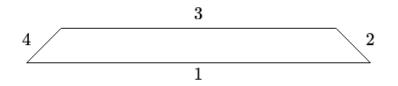
Your answer is considered correct if its absolute or relative error does not exceed 10^{-4} .

Formally, let your answer be a, and the jury's answer be b. Your answer is accepted if and only if $\frac{|a-b|}{\max(1,|b|)} \leq 10^{-4}$.

Explanation of the samples

In the first sample, the given dice is a square, so every sides have an equal chance of being the landing side.

In the second sample, the given dice is a trapezoid.



We can see that the 3-rd side has a highest probability of being the landing side. Intuitively, this is because when rolling the dice with the 2-nd or the 4-th side down, the dice will roll to the third side. This also means the 2-nd and the 4-th sides will have the probability of 0 to be the landing side.

Note

If the vertices of the dice are $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, the dice's centroid (C_x, C_y) can be computed with the following formulas:



The 2022 ICPC Asia Ho Chi Minh Regional Contest

HCMUTE - 9 December 2022



$$C_x = \frac{1}{6A} \sum_{i=1}^n (x_i + x_{(i \mod n+1)}) (x_i y_{(i \mod n+1)} - x_{(i \mod n+1)} y_i)$$
$$C_y = \frac{1}{6A} \sum_{i=1}^n (y_i + y_{(i \mod n+1)}) (x_i y_{(i \mod n+1)} - x_{(i \mod n+1)} y_i)$$

where A is the *signed* area of the polygon:

$$A = \frac{1}{2} \sum_{i=1}^{n} (x_i y_{(i \mod n+1)} - x_{(i \mod n+1)} y_i)$$

Sample Input 1	Sample Output 1
4	0.25
0 0	0.25
1 0	0.25
1 1	0.25
0 1	

Sample Input 2	Sample Output 2
4	0.46944215849887844
0 0	0
10 0	0.5305578415011215
9 1	0
1 1	