# The 2022 ICPC Asia Ho Chi Minh Regional Contest 

HCMUTE - 9 December 2022
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## Problem A <br> Aqua

Aqua is a logic puzzle game played on a rectangular board divided into $r$ rows and $c$ columns. The rows are numbered from 1 to $r$ from bottom to top, and the columns are numbered from 1 to $c$ from left to right. The cell on the $i$-th row and the $j$-th column is denoted as $(i, j)$.

The board is divided into several orthogonally connected cages using vertical and horizontal boundaries. More precisely, each cage is a non-empty subset of the board's cells. Each cell on the board belongs to exactly one cage. The set of cells of a cage forms a connected component: If two cells belong to the same cage, it is always possible to move from one cell to the other, by passing through only cells of that cage, so that each step is a move between side-adjacent cells.


Figure A.1: A board with 6 rows, 7 columns and 8 cages
In each cage, the puzzle creator pours water from top edges of the cage. Because of gravity, water falls down to the cells at lower height. He pours water so as to satisfy all below conditions:

- Every cell is either completely filled with water or completely empty.
- A cage can be full of water, completely empty or partially filled.
- In every cage, there exists some level $\ell$ such that all cells below this level of the cage are filled, and all cells above this level of the cage are empty. In other words, let $\left(r_{1}, c_{1}\right)$ and $\left(r_{2}, c_{2}\right)$ be two cells of the same cage so that $r_{1} \leq r_{2}$, if the cell $\left(r_{2}, c_{2}\right)$ is filled, then the cell $\left(r_{1}, c_{1}\right)$ must be filled as well.
- Cages are treated independently and separately. Therefore, the above rule does not apply to cells belonging to different cages.


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Below is an example of a valid board:


Figure A.2: A valid board
Below are some examples of invalid boards:


Figure A.3: An invalid board - cells $(5,5)$ and $(5,7)$ should be either both filled or both empty, since they are in the same cage and on the same row.


Figure A.4: An invalid board - cells $(4,2),(4,3)$ and $(4,4)$ are empty, but they are in the same cage as cell $(5,2)$, which is filled with water.


Figure A.5: An invalid board - cells can not be partially filled.
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The puzzle creator not only follows the above rules, he also pays attention to the number of filled cells on every row. In some rows, he wants the number of filled cells to be equal to some fixed values, while not placing any restrictions on other rows.

Your task is to count the number of ways to pour water into cages satisfying all the above rules and requirements. Two ways are considered different iff there exists a cell which is filled in one way, but is empty in the other.

## Input

The first line of the input contains 2 integers $r$ and $c(1 \leq r, c \leq 20)$. The next $r+1$ lines describe the board according to the following rules:

- Each line contains exactly $2 \cdot c+1$ characters, each is any of the following: spaces, underscores (' _') and pipes (' ${ }^{\prime}$ ).
- All characters in even index position in the first line and the last line are underscores, which demonstrate the top and the bottom borders of the board.
- Every underscore in the input, except the ones mentioned above, represents a horizontal cage boundary between two consecutive cells in the same column.
- The first and the last characters of all lines (except the first one) are pipes, which demonstrate the left and the right borders of the board.
- Every pipe in the input, except the ones mentioned above, represents a vertical cage boundary between two consecutive cells in the same row.

See samples for better understanding.
By definition:

- If there is not any horizontal cage boundary between two consecutive cells in a column, these two cells belong to the same cage.
- If there is not any vertical cage boundary betwen two consecutive cells in a row, these two cells belong to the same cage.

The last line contains $r$ integers $x_{1}, x_{2}, \ldots, x_{r}\left(-1 \leq x_{i} \leq c\right)$ describing requirements regarding the number of filled cells in each row. If $x_{i}=-1$, the $(r-i+1)$-th row can have an arbitrary number of cells with water. Otherwise, the number of filled cells in this row must be exactly $x_{i}$.

It is guaranteed that the board has at most 20 cages.

## Output

The output contains exactly one integer - the number of valid board satisfying the given conditions, modulo $10^{9}+7$.

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## Explanation of the sample

The first sample corresponds to the figure A. 2 above.

| Sample Input 1 | Sample Output 1 |
| :---: | :---: |
| 67 | 1 |
|  |  |
| \| | |_ _ | | | | |  |
| \| $\\|_{-}$ |  |
| \| | _ - | | |  |
| \| | |_ _ | | |  |
|  |  |
| 637222 |  |


| Sample Input 2 | Sample Output 2 |
| :---: | :---: |
| 23 | 3 |
| ${ }^{-}-$- |  |
| $I_{-}$- - ${ }^{\text {I }}$ |  |
| -1 -1 |  |


| Sample Input 3 | Sample Output 3 |
| :---: | :---: |
| 33 | 1 |
| - - - |  |
| $\begin{array}{lll} \mid & - & 1 \\ \text { । } & \text { \| } & \text { \| } \end{array}$ |  |
| \|_|_|_| |  |
|  |  |

