

# Median of Xor Sequence

## Problem ID: medianxor

Given four non-negative integers  $a, b, c$  and  $d$ ; let  $S$  be the “multiset” containing all values  $z = x \oplus y$  of all pairs of integers  $(x, y)$  such that  $a \leq x \leq b$  and  $c \leq y \leq d$ . Your task is to find the median value of  $S$ .

Please note that  $S$  is a “multiset”. In other words, if there are several pairs  $(x, y)$  with the same value of  $x \oplus y$ , this value appears multiple times in  $S$ .

For example, consider  $a = 3, b = 5, c = 6$  and  $d = 9$ . We have:

- $3 \oplus 6 = 5, 3 \oplus 7 = 4, 3 \oplus 8 = 11, 3 \oplus 9 = 10$
- $4 \oplus 6 = 2, 4 \oplus 7 = 3, 4 \oplus 8 = 12, 4 \oplus 9 = 13$
- $5 \oplus 6 = 3, 5 \oplus 7 = 2, 5 \oplus 8 = 13, 5 \oplus 9 = 12$

Hence, 12 elements of  $S$ , in increasing order, are 2, 2, 3, 3, 4, 5, 10, 11, 12, 12, 13, 13; meaning that the median of  $S$ , the sixth element, is 5.

A bitwise XOR (denoted as  $\oplus$ ) is a binary operation that takes two bit patterns of equal length and performs the logical exclusive OR operation on each pair of corresponding bits. The result in each position is 1 if and only if two bits are different, and is 0 if two bits are equal. For example:

- $3 \oplus 6 = 011_2 \oplus 110_2 = 101_2 = 5$
- $4 \oplus 7 = 100_2 \oplus 111_2 = 011_2 = 3$
- $5 \oplus 8 = 0101_2 \oplus 1000_2 = 1101_2 = 13$

The median value of a sequence of numbers in increasing order  $v_1 \leq v_2 \leq \dots \leq v_n$  is  $v_{\frac{n}{2}}$  if  $n$  is even and  $v_{\frac{n+1}{2}}$  if  $n$  is odd.

## Input

The first line of the input contain an integer  $t$  ( $1 \leq t \leq 75$ ) — the number of test cases.

In the last  $t$  lines, each contains four integers  $a, b, c, d$  ( $0 \leq a, b, c, d < 10^{200}, a \leq b, c \leq d$ ) representing a test case. All numbers are in decimal form.

## Output

For each test case, write a single integer on a single line denoting the median value of  $S$ . All numbers should be in decimal form.

| Sample Input 1              | Sample Output 1 |
|-----------------------------|-----------------|
| 2<br>3 5 6 9<br>11 13 20 22 | 5<br>26         |